

$$r^2 + r_1^2 - \overline{PP_1}^2 = 2r r_1 \frac{\mu r_1 (\mu r \pm \mu_1 r_1 \cos i) - \mu_1 r (\mu_1 r_1 \pm \mu r \cos i)}{\mu^2 r_1^2 - \mu_1^2 r^2}$$

ist, so ist:

$$\cos S = \frac{\mu r_1 (\mu r \pm \mu_1 r_1 \cos i) - \mu_1 r (\mu_1 r_1 \pm \mu r \cos i)}{\mu^2 r_1^2 - \mu_1^2 r^2},$$

oder auch:

$$\cos S = \frac{(\mu^2 - \mu_1^2) r r_1 \pm \mu \mu_1 (r_1^2 - r^2) \cos i}{\mu^2 r_1^2 - \mu_1^2 r^2},$$

woraus:

$$\sin S^2 = \frac{(r_1^2 - r^2) \{ \mu^2 (\mu r_1 \mp \mu_1 r \cos i)^2 - \mu_1^2 (\mu_1 r \mp \mu r_1 \cos i)^2 \}}{(\mu^2 r_1^2 - \mu_1^2 r^2)^2},$$

also ferner:

$$\text{tang } S^2 = \frac{(r_1^2 - r^2) \{ \mu^2 (\mu r_1 \mp \mu_1 r \cos i)^2 - \mu_1^2 (\mu_1 r \mp \mu r_1 \cos i)^2 \}}{\{ \mu r_1 (\mu r \pm \mu_1 r_1 \cos i) - \mu_1 r (\mu_1 r_1 \pm \mu r \cos i) \}^2}$$

folgt.

Weil

$$2 \sin \frac{1}{2} S^2 = 1 - \cos S, \quad 2 \cos \frac{1}{2} S^2 = 1 + \cos S$$

ist, so ist auch:

$$2 \sin \frac{1}{2} S^2 = \frac{(r_1 - r) \{ \mu (\mu r_1 \mp \mu_1 r \cos i) + \mu_1 (\mu_1 r \mp \mu r_1 \cos i) \}}{\mu^2 r_1^2 - \mu_1^2 r^2},$$

$$2 \cos \frac{1}{2} S^2 = \frac{(r_1 + r) \{ \mu (\mu r_1 \mp \mu_1 r \cos i) - \mu_1 (\mu_1 r \mp \mu r_1 \cos i) \}}{\mu^2 r_1^2 - \mu_1^2 r^2},$$

also:

$$\text{tang } \frac{1}{2} S = \sqrt{\frac{r_1 - r}{r_1 + r} \cdot \frac{\mu (\mu r_1 \mp \mu_1 r \cos i) + \mu_1 (\mu_1 r \mp \mu r_1 \cos i)}{\mu (\mu r_1 \mp \mu_1 r \cos i) - \mu_1 (\mu_1 r \mp \mu r_1 \cos i)}}$$

oder:

$$\text{tang } \frac{1}{2} S = \sqrt{\frac{r_1 - r}{r_1 + r} \cdot \frac{\mu^2 r_1 + \mu_1^2 r \mp \mu \mu_1 (r + r_1) \cos i}{\mu^2 r_1 - \mu_1^2 r \mp \mu \mu_1 (r - r_1) \cos i}}$$

Setzt man:

$$\text{tang } \theta = \frac{\mu_1}{\mu} \cdot \frac{\mu_1 r \mp \mu r_1 \cos i}{\mu r_1 \mp \mu_1 r \cos i},$$

so ist:

$$\text{tang } \frac{1}{2} S = \sqrt{\frac{r_1 - r}{r_1 + r} \cdot \frac{1 + \text{tang } \theta}{1 - \text{tang } \theta}},$$

also:

$$\text{tang } \frac{1}{2} S = \sqrt{\frac{r_1 - r}{r_1 + r} \text{tang } (45^\circ + \theta)};$$

und weil nach dem Obigen: